

# III Workshop on Dynamics Numeration and Tilings (III FloripaDynSys)

20-24 March, 2017  
Florianópolis, Brazil



### **Organization committee**

Ali Messaoudi	(Universidade Estadual Paulista, Brasil)	
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Vladimir Pestov	(University of Ottawa, Canada)	

SCHEDULE

Monday 20		Tuesday 21		Wednesday 22		Thursday 23		Friday 24	
08:00   09:20	Registration	09:00   10:00	Mini course 2 Gene Abrams	09:00   10:00	Mini course 2 Gene Abrams	09:00   09:30	Adam Peder Wie Sørensen	09:00   10:00	Mini course 2 Gene Abrams
09:20   10:00	Opening	10:00   10:30	Coffee break	10:00   10:30	Rodrigo Bissacot Proença	09:30   10:00	Regis Varão	10:00   10:30	Coffee break
10:00   10:30	Thierry Giordano	10:30   11:00	Artur Oscar Lopes	10:30   11:00	Ruy Exel	10:00   10:30	Coffee break	10:30   11:00	Ali Tanzibi
10:30   11:00	Alexandre Tavares Baraviera	11:00   11:30	Cristóbal Gil Canto	11:00   12:00	Poster session	10:30   11:00	Hui Li	11:00   11:30	Roozbeh Hazrat
11:00   11:30	Johan Öinert	11:30   12:00	Eduardo Garibaldi			11:00   11:30	Ping Ng	11:30   12:00	Benito Pires
11:30   12:00	Krerley De Oliveira					11:30   12:00	Samuel Senti		
					Conference Lunch				
14:00   15:00	Mini course 1 Tullio Ceccherini-Silberstein	14:00   15:00	Mini course 1 Tullio Ceccherini-Silberstein	14:00   15:00	Mini course 1 Tullio Ceccherini-Silberstein	14:00   15:00	Mini course 1 Tullio Ceccherini-Silberstein	14:00   14:30	Milton Cobo
15:00   15:30	Arek Goetz	15:00   15:30	Benjamin Steinberg	15:00   15:30		15:00   15:30	Toke M. Carlsen	14:30   15:00	Raimund Preusser
15:30   16:00	Gilles Castro	15:30   16:00	Carlos Gustavo T. De A. Moreira	15:30   16:00		15:30   16:00	Carlos Lardizabal	15:00   15:30	Vladimir Pestov
16:00   16:30	Coffee break	16:00   16:30	Coffee break	16:00   16:30		16:00   16:30	Coffee break	15:30   16:00	Closure
16:30   17:00	Miguel Abadi	16:30   17:00	Lisa Orloff Clark	16:30   17:00		16:30   17:00	Carlos Alberto Maquera Apaza		
17:00   17:30	Maria Ramirez Solano	17:00   17:30	Udayan B. Darji	17:00   17:30		17:00   17:30	Kulumani Rangaswami		

# Mini courses

## The Garden of Eden Theorem (old and new)

Tullio G. Ceccherini-Silberstein\* and Michel Coornaert

Università degli Studi del Sannio / Université de Strasbourg

- (1) Cellular automata, subshifts, amenable groups, and the GoE-T for cellular automata over amenable groups;
- (2) Entropy of compact dynamical systems, Markov partitions, and the GoE-T for toral automorphisms;
- (3) Pontryagin duality, automorphisms of compact Abelian groups, and the GoE-T.

## Minicourse on Leavitt path algebras

Gene Abrams

University of Colorado Colorado Springs

### Lecture 1: Introduction, motivation, and basic properties

In this first (of three) introductory lecture we will introduce the algebraic structures given in the title.

The general idea is this. We associate to any directed graph  $E$  and field  $K$  the  $K$ -algebra  $L_K(E)$ , called the *Leavitt path algebra of  $E$  with coefficients in  $K$* . A formal definition of  $L_K(E)$  will be given in the talk. Suffice it to say (for purposes of this abstract) that various properties of the graph  $E$  are reflected in the algebraic structure of  $L_K(E)$ , and vice versa.

The first part of this Lecture will focus on motivating why and how these Leavitt path algebras arise in a natural way. We will then present the formal definition of  $L_K(E)$ , and subsequently examine some specific examples, including the motivating examples of the so-called “Leavitt algebras” studied by W.G. Leavitt in the early 1960’s.

Once the basic ideas have been presented, we will look at some interesting (highly nontrivial) questions which arise even at this early stage. We will then look at the monoid of finitely generated projective modules over a Leavitt path algebra, a structure which has played a central role in the general theory.

## Lecture 2: Algebraic properties

In this second (of three) introductory lecture we will consider how various graph-theoretic properties of the graph  $E$  yield corresponding ring-theoretic properties in the algebra  $L_K(E)$ , and vice versa.

We will present some standard graph-theory ideas (e.g. 'directed path', 'cycle'), as well as some not-so-standard ideas (e.g. 'exit for a cycle', 'hereditary and saturated subsets of vertices'). We then show how these ideas directly affect the algebraic structure of the corresponding Leavitt path algebra. Specifically, we will consider the graph-theoretic properties of  $E$  which imply (indeed, are equivalent to) the Leavitt path algebra  $L_K(E)$  being: simple; prime; purely infinite simple; exchange; von Neumann regular; etc.

At the end of this lecture we will comment on the connection between some of these properties and the corresponding properties for the graph  $C^*$ -algebra  $C^*(E)$ .

## Lecture 3: Applications, generalizations, and future directions of research

In this third (of three) introductory lecture we will give a (necessarily brief) overview of various aspects of the subject.

First, we will present some of the ways in which Leavitt path algebras themselves, and some of the tools developed to study them, have been used to establish results in areas outside the realm of Leavitt path algebras per se.

Second, we will present a few of the many ways that Leavitt path algebras have been generalized and modified, including (but not limited to): the groupoid approach; separated graphs; higher rank graphs; the graded structure; weighted graphs; etc.

Finally, we will conclude the lecture (and Minicourse) by presenting some (currently) open questions in the subject.

Many of the III FloripaDynSys attendees have made major contributions some or all of these three aspects of Leavitt path algebras.

# Talks

## The complete classification of unital graph algebras

Adam P. W. Sørensen

University of Oslo, Norway.

I will discuss recent joint work with Eilers, Restorff, and Ruiz that gives a complete classification of unital graph algebras. I will explain the progression that brought us from purely infinite Cuntz-Krieger algebras to all unital graph algebras, and the new ideas we needed for the final push.

## Approximation of the top Lyapunov exponent for certain random cocycles

Alexandre Baraviera

UFRGS, Brazil.

Consider a compact metric space  $(\Sigma, d)$  and the corresponding space of sequences  $\Omega := \Sigma^{\mathbb{Z}}$ , where the homeomorphism  $T\{\omega_i\} := \{\omega_{i+1}\}$ , known as the full shift map, is defined. Given a probability measure  $\mu$  defined on  $\Sigma$ , the product measure  $\mathbb{P} := \mu^{\mathbb{Z}}$  is an ergodic  $T$ -invariant measure.

Take a continuous function  $A: \Sigma \rightarrow GL_d(\mathbb{R})$ . A *random cocycle* is the map  $F_{(A, \mu)}: \Sigma \times \mathbb{R}^d \rightarrow \Sigma \times \mathbb{R}^d$  defined as

$$F(\omega, v) = (T\omega, A(x).v)$$

Then the top Lyapunov exponent is defined as the number

$$\gamma_+(A) = \lim_{n \rightarrow \infty} \frac{1}{n} \log \|A(T^{n-1}\omega) \cdots A(T\omega)A(\omega)\|$$

that exists  $\mathbb{P}$ -almost surely (Furstenberg-Kesten).

The top Lyapunov exponent can be obtained in this context by means of a formula due to Furstenberg:

$$\gamma_+(A) = \int_{\Sigma} \int_{\mathbb{P}(\mathbb{R}^d)} \log \|A(x)p\| d\nu(p) d\mu(x)$$

where  $\nu$  is a stationary measure on the projective space  $\mathbb{P}(\mathbb{R}^d)$



An interesting question is to obtain a good approximation of the top Lyapunov exponent. Here we present an idea to obtain this number (with an estimate for the error) based on an approximation of the transfer operator used to construct the stationary measure. With this approximation we also obtain another proof of the continuous dependence of the top Lyapunov exponent as a function of  $A$ , a well known result from E. Le Page.

This is a joint work with Pedro Duarte (Univ. de Lisboa).

## **Invariance principle and rigidity of high entropy measures**

Ali Tahzibi

USP, Brazil

A deep analysis of Lyapunov exponents of stationary sequence of matrices going back to Furstenberg, for more general linear cocycles by Ledrappier and generalized to the context of non linear cocycles by Avila and Viana gives an invariance principle for invariant measures with vanishing central exponents. In a joint work with J. Yang we give a new criterium formulated in terms of entropy for the invariance principle and in particular obtain a simpler proof for some of the known invariance principle results.

As a byproduct, we study ergodic measures of partially hyperbolic diffeomorphisms whose center foliation is 1-dimensional and forms a circle bundle. We show that for any such  $C^2$  diffeomorphism which is accessible, weak hyperbolicity of ergodic measures of high entropy implies that the system itself is of rotation type. As mentioned to us by Sylvain Crovisier, our result may be also used to give a more precise information on the results of Diaz-Gelfert-Rams where they study transitive step skew product maps modeled over a complete shift whose fiber maps are circle maps.

## **Two dimensional interval exchange transformations: Piecewise Isometries.**

Arek Goetz

San Francisco State University, USA.

We offer a glimpse of how rich the dynamics of piecewise isometries is, especially as seen from computer graphics. We survey some recurrence/rescaling results and techniques for a pizza map. A pizza map is an invertible map of the plane that acts as first a permutation of cones that partition the plane followed by a translation. Investigation has been joint mainly with Peter Ashwin from Exeter, UK.

## **The KMS Condition for the homoclinic equivalence relation and equilibrium probabilities**

Artur O. Lopes

UFRGS, Brazil

D. Ruelle consider a general setting where he is able to describe a formulation for the concept of Gibbs states based on conjugating homeomorphism in the so called Smale spaces. The Anosov diffeomorphisms are on this class. On this setting he shows a relation of KMS states of  $C^*$ -algebras and equilibrium probabilities of Thermodynamic Formalism. We will present a shorter proof of this equivalence on the symbolic space  $\{1, 2, \dots, d\}^{\mathbb{Z}}$ . Our purpose is to present the basic facts of  $C^*$ -Algebras on a language which is more close to the one used on Ergodic Theory. This is a joint work with Gabriel Mantovani.

## **Topological transitivity and mixing of the composition operator on $L^p$ -spaces**

Udayan Darji and Benito Pires\*

University of Louisville - USA / University of São Paulo - Brazil

Let  $X = (X, \Sigma, \mu)$  be a  $\sigma$ -finite measure space and  $f : X \rightarrow X$  be an one-to-one bimeasurable transformation satisfying  $\mu(f(B)) \geq c_1 \mu(B)$  for some constant  $c_1 > 0$  and every measurable set  $B$ , then  $T_f : \varphi \mapsto \varphi \circ f$  is a bounded linear operator acting on  $L^p(X, \Sigma, \mu)$ ,  $1 \leq p < \infty$ , called the *composition operator* induced by  $f$ . We provide necessary

and sufficient conditions on  $f$  for  $T_f$  to be topologically transitive or topologically mixing. We also give two examples of one-to-one bimeasurable transformations whose composition operators are topologically transitive but not topologically mixing. Finally, we show that the composition operator induced by a bi-Lipschitz  $\mu$ -contraction (or more generally, by a  $\mu$ -dissipative transformation) defined on a finite measure space is always topologically mixing.

## Étale groupoid algebras

Benjamin Steinberg

City College of New York, USA

Over the past several years étale groupoid algebras over arbitrary commutative rings have played a unifying role in connecting results about associative algebras, e.g. Leavitt path algebras, with results in operator algebras, e.g., Cuntz-Krieger algebras. In this talk we survey some of the results and research directions and discuss the role of inverse semigroup theory, which is often hidden in the background.

## Operational partitions of unity, entropies and quantum walks

Carlos F. Lardizabal

Universidade Federal do Rio Grande do Sul (UFRGS), Brazil.

The quantum dynamical entropy developed by Alicki and Fannes is motivated by an earlier approach of Lindblad to the non-commutative generalization of the Kolmogorov-Sinai entropy and considers the description of  $C^*$ -quantum dynamical systems by means of quantum symbolic models. In this introductory talk we review such model in the context of quantum walks and its communicating classes. No previous knowledge of quantum mechanics is assumed.

## The Markov and Lagrange spectra and dynamical generalizations

Carlos Gustavo T. de A. Moreira

IMPA, Brazil

The classical Markov and Lagrange spectra are sets of real numbers related to Diophantine Approximations. We will present classical and recent results on these sets involving their dynamical characterization and aspects of fractal geometry. We will discuss natural generalizations of these spectra in the context of Dynamical Systems and Differential Geometry, and recent results related to these generalizations, in collaboration with Rom  a, Cerqueira and Matheus.

## Grupos de lie nilpotentes agindo hiperbolicamente

Carlos Maquera

USP, Brazil

Um grupo de Lie  $G$  age como uma *a   o hiperb  lica* numa variedade  $M$ , se existe um elemento de  $G$  que age como difeomorfismo parcialmente hiperb  lico cuja dire   o central    a dire   o tangente   s   rbitas da a   o. Quando um dos fibrados est  vel ou inst  vel da decomposi   o hiperb  lica for de dimens  o um, dizemos que a a   o    de *codimens  o um*. Mostramos que se um grupo de Lie nilpotente  $G$  age como uma a   o hiperb  lica de codimens  o um, ent  o  $G$     uma extens  o por um  $n$ -toro do grupo  $\mathbb{R}^k$ . Em particular: Todo grupo de Lie nilpotente que age de maneira fiel e hiperb  lica de codimens  o um    abeliano.

## The cycline subalgebra of a Kumjian-Pask algebra

Lisa Orloff Clark, Crist  bal Gil Canto\* and Alireza Nasr-Isfahani

University of Otago, New Zealand. / University of M  laga, Spain. / University of Isfahan, Iran.

We study analogues of Leavitt path algebras associated to higher-rank graphs in which paths have a  $k$ -dimensional degree and a 1-graph reduces to a directed graph. These algebras are called Kumjian-Pask

algebras. Let  $\Lambda$  be a row-finite higher-rank graph with no sources. We identify a maximal commutative subalgebra  $\mathcal{M}$  inside the Kumjian-Pask algebra  $KP_R(\Lambda)$ . Analogously to the 1-graph algebra subject, a central topic in  $k$ -graphs algebras is to determine when a given homomorphism from  $KP_R(\Lambda)$  is injective; this is the content of the uniqueness theorems. We also prove a generalized Cuntz-Krieger uniqueness theorem for Kumjian-Pask algebras which says that a representation of  $KP_R(\Lambda)$  is injective if and only if it is injective on  $\mathcal{M}$ .

## **An alphabetical approach to the Nivat's conjecture**

Eduardo Garibaldi

UNICAMP, Brazil

Nivat's conjecture claims that only periodic configurations on a two-dimensional integer lattice may satisfy a low complexity assumption. Since techniques used to address the Nivat's conjecture usually relies on Morse-Hedlund Theorem, an improved version of this classical result may mean a new step towards a proof for the conjecture. In this talk, we discuss how, following methods highlighted by Cyr and Kra, an extension of the so far best known result to the Nivat's conjecture may be derived from an alphabetical version of Morse-Hedlund Theorem. This a joint work with C. Colle.

## **The boundary path space of a labelled graph**

Gilles G. de Castro\*, Giuliano Boava and Fernando de L. Mortari

UFSC, Brazil. / UFSC, Brazil. / UFSC, Brazil.

The notion of  $C^*$ -algebras of labelled graphs was developed by Bates and Pask. Such algebras generalize, among others, Cuntz-Krieger algebras, Exel-Laca algebras and graph algebras. These algebras contain a commutative  $C^*$ -subalgebra called the diagonal subalgebra. By using Exel's framework on how to construct a  $C^*$ -algebra from an inverse semigroup in this context, we can describe the spectrum of the diagonal subalgebra. The space obtained is a generalization of the boundary

path space of a graph, which is a generalization of the one-sided edge shift of a graph.

## **Branching systems of higher-rank Graphs**

Hui Li

University of Windsor, Canada

In this talk, I will introduce the notion of branching systems of  $k$ -graphs. In particular, I will show how to concretely construct a branching system of a row-finite 2-graph without sources. This is joint work with Professor Daniel Gonçalves and Professor Danilo Royer.

## **Epsilon-strongly graded rings and crossed products by twisted partial actions**

Patrik Nystedt, Johan Öinert\* and Héctor Pinedo

University West, Sweden / Blekinge Institute of Technology, Sweden / Industrial University of Santander, Colombia

This talk is based on a joint work with Patrik Nystedt and Héctor Pinedo.

Epsilon-strongly group graded rings constitute a class of rings which contains all strongly group graded rings and all crossed products by unital twisted partial group actions. A result of Năstăsescu, Van den Bergh and Van Oystaeyen (1989) gives a characterization of strongly group graded rings which are separable over their canonical 'degree zero' subrings. A more recent result of Bagio, Lazzarin and Paques (2010) gives a characterization of certain crossed products, by unital twisted partial group actions, which are separable over their coefficient subrings. We are able to simultaneously generalize both of these results by giving a characterization of separable epsilon-strongly group graded rings. We also provide an example of a separable epsilon-strongly group graded ring (not strongly graded!) which answers a question of Le Bruyn, Van den Bergh and Van Oystaeyen (1988).

## Equilibrium States for Random Local Diffeomorphisms

Krerley Oliveira

Federal University of Alagoas, Brazil

In this talk we discuss a recent results on uniqueness of equilibrium states for some Iterate Function Systems (IFS).

We prove existence of relative maximal entropy measures for certain random dynamical systems that are skew products of the type  $F(x, y) = (\theta(x), f_x(y))$ , where  $\theta$  is an invertible map preserving an ergodic measure  $\mathbb{P}$  on a Polish space and  $f_x$  is a local diffeomorphism of a compact Riemannian manifold exhibiting some non-uniform expansion. As a consequence of our proofs, we obtain an integral formula for the relative topological entropy as the integral the of logarithm of the topological degree of  $f_x$  with respect to  $\mathbb{P}$ . When  $F$  is topologically exact and the supremum of the topological degree of  $f_x$  is finite, the maximizing measure is unique and positive on open sets.

## On Irreducible representations of Leavitt Path Algebras

Kulumani M. Rangaswamy

University of Colorado, USA

This talk will consider the Isomorphism Classes of Irreducible Representations of Leavitt path algebras of arbitrary directed graphs and their cardinality. We will describe Leavitt path algebras whose irreducible representations have special properties such as being injective,  $\sigma$ -injective and graded etc. Finally, a subclass of Leavitt path algebras with bounded index of nilpotence are also described.

## Using Steinberg algebras to study Leavitt path algebras

Lisa Orloff Clark

University of Otago, New Zealand

The Leavitt path algebra associated to a graph  $E$  can be viewed abstractly as the universal algebra generated by a Leavitt  $E$ -family. There are several concrete models of a given Leavitt path algebra. The most common model views an LPA as a particular quotient of the free algebra generated by the vertices and edges. The Stienberg algebra associated to the graph groupoid  $G_E$  is an alternate model. In this talk, I will describe some recent results that demonstrate the power of the Steinberg algebra model.

## On the K-theory of $C^*$ -algebras for substitution tilings

Maria Ramirez-Solano\* and Daniel Gonçalves

University of Southern Denmark, Denmark. / Universidade Federal de Santa Catarina, Brazil.

Under suitable conditions, a substitution tiling gives rise to a Smale space, from which three equivalence relations can be constructed, namely the stable, unstable, and asymptotic equivalence relations. In this talk I will give the construction of a cochain map of a cochain complex to itself. Taking the cohomology (resp. homology of the dual) of the cochain map and of the chain complex we can compute the K-theory for the  $C^*$ -algebra of the stable (resp. unstable) equivalence relation for tilings of the line and of the plane. Moreover, we provide formulas to compute the  $K$ -theory of the  $C^*$ -algebras of these 3 equivalence relations.

## Shortest path distribution and prime decomposition

Miguel Abadi\* and Rodrigo Lambert

IME - USP, Brazil. / FaMat - UFU, Brazil.

For a symbolic dynamical system and for each natural  $n$ , we introduce the Shortest Path function between two  $n$ -cylinders (or sequences of size  $n$ ). This function describes the underlying graph of connections of the dynamical system. Concentration, fluctuation and large deviation were proven in a general ergodic context. We focus on the asymptotic distribution of the shortest path function and show its



link to the prime decomposition of natural numbers. The asymptotic mean of this function also presents interesting behaviour related to the complexity of the process. We illustrate with a uniform independent measure over a two symbols alphabet

## **Wandering intervals for affine perturbations of the Arnoux-Yoccoz family**

Milton Cobo

Universidade Federal do Espírito Santo, Brazil.

Remarkably Interval Exchange Maps (IEMs) appear in many different contexts in dynamical systems. Although the dynamic of individual IEMs is rather simple, some renormalizations in the world of IEMs play a significant role in many problems (the Rauzy-Zorich renormalization). In contrast very little is known about the structure of the world of Affine Interval Exchange Maps (AIEMs). These are the simplest perturbations of IEMs, where the slopes are allowed to be different from one (they are not isometries).

We consider the famous Arnoux-Yoccoz family of IEMs, a source of examples and counter-examples in the field. We show that some specific perturbations of these maps yield AIEMs with wandering intervals, where the minimal invariant set is a Cantor set. All of this is related to some fractal sets of the plane (the Rauzy fractals and their dual counterparts).

These problems are closely related to  $\beta$ -Numeration Systems of the plane where the base of numeration is an algebraic complex number.

We will also indicate possible applications of this work.

This is a joint work with Rodolfo Gutierrez and Alejandro Maass of CMM (Univ. of Chile).

## **Purely infinite corona algebras and extensions**

Ping W Ng

U. of Louisiana at Lafayette, USA

We classify all extensions of the form

$$0 \rightarrow B \rightarrow E \rightarrow C(X) \rightarrow 0$$

where  $B$  is a nonunital separable simple finite real rank zero  $\mathbb{Z}$ -stable  $C^*$ -algebra with continuous scale, and  $X$  is a finite CW complex.

## Normal forms for weighted Leavitt path algebras

Raimund Preusser

Universidade de Brasília, Brazil.

This talk is based on joint work with R. Hazrat. Weighted Leavitt path algebras (wLpas) are a generalisation of Leavitt path algebras and cover the algebras  $L_K(n, n+k)$  constructed by Leavitt. Using Bergman's diamond lemma, we give normal forms for elements of a wLPA. Using the normal form we classify the wLpas which are domains, simple and graded simple rings. For a large class of wLPAs we establish a local valuation and as a consequence we prove that these algebras are prime, semiprimitive and nonsingular but contrary to Leavitt path algebras, they are not graded von Neumann regular.

## Rigidity for partially hyperbolic diffeomorphisms

Régis Varão

UNICAMP, Brazil.

In this work we completely classify smooth conjugacy for conservative partially hyperbolic diffeomorphisms homotopic to a linear Anosov automorphism on the 3-torus by its center foliation behavior. We prove that the uniform version of absolute continuity for the center foliation is the natural hypothesis to obtain  $C$  conjugacy to its linear Anosov automorphism. On a recent work Avila, Viana and Wilkinson proved that for a perturbation in the volume preserving case of the time-one map of an Anosov flow absolute continuity of the center foliation implies smooth rigidity. The absolute version of absolute continuity is

the appropriate scenario for our context since it is not possible to obtain an analogous result of Avila, Viana and Wilkinson for our class of maps, for absolute continuity alone fails miserably to imply smooth rigidity for our class of maps. Our theorem is a global rigidity result as we do not assume the diffeomorphism to be at some distance from the linear Anosov automorphism. We also do not assume ergodicity. In particular a metric condition on the center foliation implies ergodicity and smooth center foliation.

## **Stability of the Phase Diagram on Gibbs Theory**

Rodrigo Bissacot

University of So Paulo (USP), Brazil.

We discuss the stability of the phase transition phenomenon on Gibbs Theory. In particular, results for ferromagnetic Ising Models and perturbations with spatially dependent external fields.

## **Classification of Leavitt path algebras via graded K-theory, a review**

Roozbeh Hazrat

Western Sydney University, Australia

There is a conjecture that graded Grothendieck groups would classify Leavitt path algebras. We review this conjecture and results obtained so far.

## **Inverse hulls for 0-left cancellative semigroups with applications to subshifts**

Ruy Exel

UFSC, Brazil

Given a 0-left cancellative semigroup  $S$ , consider its inverse hull  $H(S)$ , as well as the idempotent semi-lattice of the latter, which we denote by  $E(S)$ . Various interesting subsets of the spectrum of  $E(S)$  will be presented, each one allowing for a different reduction of the universal groupoid of  $H(S)$ . In the case of the language semigroup associated to a subshift, these reduced groupoids lead to certain interesting  $C^*$ -algebras which have been studied by Matsumoto and by Carlsen-Matsumoto. Our approach therefore shines some new light on the subtle differences between these  $C^*$ -algebras. This talk is based on joint work with Benjamin Steinberg, who will also speak on the subject of inverse hulls.

## Thermodynamical Formalism for Hyperbolic Towers

Samuel Senti

UFRJ, Brazil

In this joint work with Y. Pesin and K. Zhang we show how to establish the existence and uniqueness of equilibrium measures for multidimensional non-uniformly hyperbolic systems, as well as some statistical properties such as decay of correlations and the validity of the central limit theorem.

We will not assume any prior knowledge of the theory, and hope to spend more time on motivation and examples than on the technical aspects of the proof, albeit it is precisely in these aspects that all the work lies.

## An interesting class of free minimal actions on the Cantor set: the $\mathbb{Z}^d$ -Odometers.

Thierry Giordano\*, Ian Putnam and C. Skau

University of Ottawa, Canada. / University of Victoria, Canada. / NTNU, Norway.

$\mathbb{Z}$ -odometers are a very rigid class of minimal Cantor systems: any two orbit equivalent  $\mathbb{Z}$ -odometers are conjugate. The situation is totally different in higher dimensions.

In this talk, I will review the construction of  $\mathbb{Z}^d$ -odometers and describe several results recently proved on this class of free minimal actions on the Cantor set. I will present two equivalent descriptions of  $\mathbb{Z}^d$ -odometers and compute their first group of cohomology. Examples of orbit equivalent  $\mathbb{Z}^2$ -odometers which are not conjugate will be presented!

## Graph algebras and groupoids

Toke M. Carlsen

University of the Faroe Islands, Faroe Islands.

I will give an overview of the recent progression in the study of the interconnection between graph  $C^*$ -algebras, Leavitt path algebras, and graph groupoids, which have lead to the following two results.

- (1) Two Cuntz–Krieger algebras  $\mathcal{O}_A$  and  $\mathcal{O}_B$  are stably isomorphic by a diagonal-preserving isomorphism if and only if the corresponding two-sided shift spaces  $\overline{X}_A$  and  $\overline{X}_B$  are flow equivalent.
- (2) Two Cuntz–Krieger algebras  $\mathcal{O}_A$  and  $\mathcal{O}_B$  are stably isomorphic by a diagonal-preserving isomorphism that intertwines the gauge actions if and only if the corresponding two-sided shift spaces  $\overline{X}_A$  and  $\overline{X}_B$  are conjugate.

## Local Entropy Theory and Complicated Continua

U. B. Darji

University of Louisville, USA.

Local entropy theory is a powerful tool which has been used to solve some problems in dynamics of continuum theory. In this talk, we give an overview of local entropy theory and its applications to continuum theory. We also state some open problems which may be resolved by local entropy theory.

## On the Schneider–Thom amenability criteria for general topological groups and some applications

Vladimir Pestov

University of Ottawa, Canada *and* UFSC, Brazil

Friedrich Martin Schneider and Andreas Thom ([arXiv:1608.08185 \[math.GR\]](#)) have recently extended in a remarkable way two most prominent classical criteria of amenability of groups — Følner’s criterion and Reiter’s property ( $P1$ ) — to the arbitrary topological groups. Already in the non-discrete locally compact case, the resulting criteria are new. In view of the speaker, it is one of the most important developments in the area of “infinite-dimensional groups” during the past decade. We will survey their results and discuss some applications of the new criteria, such as a solution of a problem of amenability of groups of measurable maps (joint with F.M. Schneider, [arXiv:1701.00281 \[math.FA\]](#)).

# Posters

## **Transitivity of linear operators**

Ali Messaoudi

UNESP, Brazil

In this lecture, we will study dynamical properties of infinite matrices associated to countable Markov chains. In particular, we will prove that a large class of these matrices are transitive and Devaney chaotic.

## **Convolution of shift-invariant probability measures in $(\mathbb{Z}/p)^{\mathbb{N}}$**

Bruno Brogni Uggioni\* and Alexandre Tavares Baraviera

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We say, generally, that a sequence of measures  $\eta_n$  converges in convolution to a measure  $\eta$  if  $\eta_n * \dots * \eta_1 \rightarrow \eta$  in the weak\* topology. Lindenstrauss has shown that, with some special conditions on the entropy of a sequence of  $p$ -invariant (and ergodic) measures  $\eta_n$  on the circle ( $\mathbb{S}^1$ ),  $\eta_n$  converges in convolution to the Lebesgue measure. In this talk, we would like to present a similar theorem to this Lindenstrauss' result, not in the circle anymore, but in the symbolic space. The theory of characters of compact and/or finite abelian groups was one of the main tools we used to prove our results. These results were obtained during my doctorate at UFRGS, under the supervision of Prof. Dr. Alexandre Tavares Baraviera.

## **Birth of limit cycles bifurcating from a nonsmooth center**

Claudio Buzzi

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This work is concerned with a codimension analysis of a two-fold singularity of piecewise smooth planar vector fields, when it behaves itself like a center of smooth vector fields (also called nondegenerate  $\Sigma$ -center). We prove that any nondegenerate  $\Sigma$ -center is  $\Sigma$ -equivalent



to a particular normal form  $Z_0$ . Given a positive integer number  $k$  we explicitly construct families of piecewise smooth vector fields emerging from  $Z_0$  that have  $k$  hyperbolic limit cycles bifurcating from the non-degenerate  $\Sigma$ -center of  $Z_0$  (the same holds for  $k = \infty$ ). Moreover, we also exhibit families of piecewise smooth vector fields of codimension  $k$  emerging from  $Z_0$ . As a consequence we prove that  $Z_0$  has infinite codimension.

It is a joint work with Marco Antonio Teixeira (Unicamp) and Tiago de Carvalho (Unesp - Bauru).

## Phase transition on Ising models with inhomogeneous external fields

Eric Ossami Endo

University of Sao Paulo, Brazil and University of Groningen, Netherlands.

The nearest-neighbours Ising model on the lattice  $\mathbb{Z}^d$  with ferromagnetic interaction is one of the most studied models in statistical mechanics. One of the main problems is to prove the existence of the phase transitions, which means that there is more than one Gibbs measure at some temperature. When the Ising model is considered on the lattice  $T^d$ , the  $d + 1$  regular on Cayley tree, Preston proved the existence of the phase transition at low temperatures for the model if we consider small positive external fields. We discuss in this talk the analogous question already studied by Bissacot, Cassandro, Cioletti and Presutti in a recent paper in (CMP-2015) for Ising models in  $\mathbb{Z}^d$  ( $d \geq 2$ ). Consider the Ising model on a Cayley tree with external fields  $h_i = h_c + \epsilon_i$ , where  $i$  is a vertex of the tree and  $h_c > 0$  is the critical external fields from Preston. We are going to present that there exists a condition for the family  $\epsilon_i > 0$  which separates and characterize the regions which we have the presence and the absence of the phase transition. The proof relies on the study of the decaying of the family  $\epsilon_i$  and the iterated maps which present a saddle-node fixed point. We will also to discuss the unidimensional Dyson models on the lattice  $\mathbb{Z}$  with the coupling constant  $J_{i,j} = |i - j|^{-\alpha}$  and external fields  $h_i = |i|^{-\gamma}$ , and to present some results to show the presence of phase transition.

## Structure and classification of generalised Bunce–Deddens algebras and their KMS states

James Rout

University of Wollongong, Australia

In this poster presentation, we present work about David Kribs and Baruch Solel’s graph theoretic generalisation of the Bunce–Deddens algebras. These  $C^*$ -algebras are constructed as the direct limits of graph  $C^*$ -algebras. We present answers for Kribs and Solel’s open questions about the simplicity and classification for large classes of generalised Bunce–Deddens algebras. We also compute the KMS states for these  $C^*$ -algebras and their Toeplitz extensions. These results are from the author’s PhD thesis supervised by Aidan Sims and Dave Robertson.

## A class of cubic Rauzy fractals

J. Bastos\*, A. Messaoudi, T. Rodrigues and D. Smania

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Consider  $A = \{1, 2, 3, 4\}$  and  $\sigma : A \longrightarrow A^*$  given by

$$\sigma(1) = 1^{a-1}2, \sigma(2) = 1^{a-1}3, \sigma(3) = 4, \sigma(4) = 1, a \geq 2.$$

The Rauzy Fractal associated to  $\sigma$  is the set

$$\mathcal{R}_a = \left\{ \sum_{i=2}^{+\infty} a_i \alpha^i, a_i a_{i-1} a_{i-2} a_{i-3} <_{lex} (a-1)(a-1)01, i \geq 5 \right\}$$

where  $\alpha$  is a complex root of  $x^3 - ax^2 + x - 1$  and  $a_i \in \{0, 1, \dots, a-1\}$ . This set is a compact and connected subset of  $\mathbb{C}$  and induces a periodic tiling of the plane.

In this work we study arithmetical and topological properties of the fractals  $\mathcal{R}_a$ . In particular we prove that the number of neighbors of  $\mathcal{R}_a$  in the periodic tiling is equal 8. We also gives explicitly an automaton that generates its boundary and, as a consequence, we prove that  $\mathcal{R}_2$  is homeomorphic to a topological disk.

## An introduction to sofic groupoids

Luiz Cordeiro

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Sofic groupoids are those which can be approximated, in a suitable sense, by finite groupoids. The class of sofic groupoids has been extensively studied in recent years due to its generality and to the fact that several conjectures, mainly in the group and equivalence relations cases, have been answered positively for them.

We will motivate and present an elementary description of sofic groupoids, some new results regarding permanence properties for this class, and provide a simpler description in the non-singular case.

## Wandering intervals for affine perturbations of the Arnoux-Yoccoz family

Milton Cobo

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Remarkably Interval Exchange Maps (IEMs) appear in many different contexts in dynamical systems. Although the dynamic of individual IEMs is rather simple, some renormalizations in the world of IEMs play a significant role in many problems (the Rauzy-Zorich renormalization). In contrast very little is known about the structure of the world of Affine Interval Exchange Maps (AIEMs). These are the simplest perturbations of IEMs, where the slopes are allowed to be different from one (they are not isometries).

We consider the famous Arnoux-Yoccoz family of IEMs, a source of examples and counter-examples in the field. We show that some specific perturbations of these maps yield AIEMs with wandering intervals, where the minimal invariant set is a Cantor set. All of this is related to some fractal sets of the plane (the Rauzy fractals and their dual counterparts).

These problems are closely related to  $\beta$ -Numeration Systems of the plane where the base of numeration is an algebraic complex number.

We will also indicate possible applications of this work.

This is a joint work with Rodolfo Gutierrez and Alejandro Maass of CMM (Univ. of Chile).

## A parametrization for a class of rauzy fractal

Jefferson Bastos and Tatiana Miguel Rodrigues de Souza\*

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In this work, we study a class of Rauzy fractals  $\mathcal{R}_a$  given by the polynomial  $x^3 - ax^2 + x - 1$  where  $a \geq 2$  is an integer. In particular we give explicitly an automaton that generates the boundary of  $\mathcal{R}_a$  and using an exotic numeration system we prove that  $\mathcal{R}_a$  is homeomorphic to a topological disk.